Theory of Double Hard Diffraction

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Abstract

In this review talk I consider the physics of rapidity gaps between two jets at hadron colliders, as a preliminary investigation toward understanding the production of a Higgs boson via weak boson fusion at the LHC.

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1 Introduction

1.1 The experiments

By double hard diffraction it is meant the production of two or more jets with a rapidity gap between two jets, where a rapidity gap is a region in (pseudo)-rapidity where no hadrons are produced above a threshold μ . Double hard diffraction has been analysed by the CDF and D0 Collaborations [1, 2, 3] at the Tevatron $p\bar{p}$ collider, operating at $\sqrt{s} = 1.8$ TeV, and in photoproduction events by the Zeus Collaboration at the ep HERA collider [4], operating at $\sqrt{s} \simeq 300$ GeV. The main difference with respect to single hard diffraction is the momentum transfer t; while in the latter t is usually much less than 1 GeV², in double hard diffraction it is very large ($|t| \gtrsim 10^3 \,\text{GeV}^2$ in the Tevatron experiments [1, 2, 3] and $|t| \gtrsim 30 \,\text{GeV}^2$ at HERA [4]).

In the experiments the events are ranked according to the rapidity interval $\Delta \eta$ between the two leading E_{\perp} jets, and measure the fraction of these events which have a gap of width $\Delta \eta_c = \Delta \eta - 2R$, with R the jet-cone size, between the leading E_{\perp} jets. One observes [1, 3, 4] a falloff of the gap fraction as the gap width $\Delta \eta_c$ increases, as expected if the gaps are merely due to multiplicity fluctuations. However, when $\Delta \eta_c \gtrsim 2$ the gap fraction becomes basically independent of $\Delta \eta_c$. Estimating the contribution of the multiplicity fluctuations by fitting the high-multiplicity data through a (double) negative binomial distribution (NBD), the D0 Collaboration [3] puts the value of the gap fraction in excess of the background at

$$f_g = 1.07 \pm 0.10 \text{ (stat)} +0.25 -0.13 \text{ (syst)}\%.$$
 (1)

The gap fraction in excess at HERA has been estimated to be about 7% [4].

1.2 The motivation

The original, and still the main, motivation for analysing double hard diffraction events is to look for a clean signal for the production of a heavy Higgs boson at hadron supercolliders [5, 6]. A heavy Higgs boson is mainly produced via gluon fusion, $gg \to H$, mediated by a top-quark loop, Fig. 1(a). The Higgs boson then decays mainly into a

pair of W or Z bosons. Such a signal, though, is going to be swamped by the WW QCD and the $t\bar{t}$ backgrounds, Fig. 2. Higgs-boson production via weak-boson fusion, WW, $ZZ \to H$, Fig. 1(b), has a smaller rate [7] but would have the distinctive feature of a rapidity gap in parton production since no color is exchanged between the quarks emitting the weak bosons. Producing a rapidity gap at the parton level is not enough though, since the gap is usually filled by soft hadrons produced in the rescattering between the spectators partons in the underlying event. In addition, the signal might be faked by the exchange of gluons in a color-singlet configuration [6]. Hence the idea to test the physics of rapidity gaps between jets at the Tevatron.

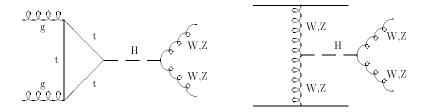


Figure 1: Higgs-boson production via (a) gluon fusion and (b) weak-boson fusion.

2 Double hard diffraction

2.1 Gaps at the parton level

A rapidity gap in parton-parton scattering at the Tevatron may be produced via γ , W, Z-boson exchange in the crossed channel, \hat{t} . However, this process is concealed by gluon exchange, which is an $\mathcal{O}(\alpha_s^2)$ process and whose rate is bigger by 2-3 orders of magnitude [6, 8]. Gluon exchange in the \hat{t} channel is likely not to produce a gap because the exchanged gluon being a color octet radiates off more gluons. Indeed while in parton-parton scattering with electroweak exchange additional gluon bremsstrahlung is mainly in the forward directions, in gluon exchange it occurs mainly in the central-rapidity region, thus filling the gap [9].

A gap may also be produced by exchanging two gluons in the \hat{t} channel in a color-singlet configuration. This is an $\mathcal{O}(\alpha_s^4)$ process, whose rate Bjorken [6] estimated to be about 10% of the one-gluon exchange rate, $\hat{\sigma}_{sing}/\hat{\sigma}_{oct} \sim 0.1$. The integral of the gluon loop formed by the two-gluon exchange is dominated by small transverse momenta. Thus the exchange is given by a hard and a soft gluon. It is not clear though if in this way a color singlet may be formed because the hard gluon has time to emit more gluons, the soft gluon that closes the loop being emitted much later (or much earlier) [10].

In the limit of high squared parton c.m. energy \hat{s} and fixed \hat{t} , an $O(\alpha_s^5)$ analysis of the gluon-bremsstrahlung pattern for parton-parton scattering with two-gluon exchange in a color-singlet configuration shows that if the transverse momentum $p_{\perp rad}$ of the radiated gluon is of the same order as the transverse momenta $p_{\perp jet}$ of the tagging jets then the gluon is radiated mainly in the central-rapidity region, like in the one-gluon exchange case; if $p_{\perp rad} \ll p_{\perp jet}$, then the gluon is radiated mainly in the forward direction, i.e. the radiation pattern is similar to the one of electroweak exchange [11]. In other words, if the bremsstrahlung gluon is hard it has a short wavelength and may resolve the color structure of the two-gluon exchange; if it is soft its resolving power is low and sees the two exchanged gluons as a color singlet.

In the limit of $\hat{s} \gg |\hat{t}|$ it is possible to resum the leading logarithmic contributions, in $\ln(\hat{s}/|\hat{t}|)$, to a scattering amplitude to all orders in α_s by using the Balitsky-Fadin-Kuraev-Lipatov (BFKL) theory [12]. Resumming the leading virtual radiative corrections to one-gluon exchange in gluon-gluon scattering, we obtain [13, 14, 15],

$$\frac{d\hat{\sigma}_{oct}}{d\hat{t}} \simeq \frac{9\pi\alpha_s^2}{2\hat{t}^2} \exp\left(-\frac{3\alpha_s}{\pi} \ln\frac{\hat{s}}{|\hat{t}|} \ln\frac{p_{\perp}^2}{\mu^2}\right),\tag{2}$$

with p_{\perp} the transverse momentum of the outgoing gluons, $\hat{t} \simeq -p_{\perp}^2$, and μ a cutoff that regulates the infrared divergence, which is there because we have not included the real emissions. Eq. (2) defines the scattering as elastic if no soft gluons with $p_{\perp} \gtrsim \mu$ appear in the final state. The exponential of eq. (2) vanishes as $\mu \to 0$, which is a general feature of bremsstrahlung emission known as infrared catastrophe or Block-Nordsieck mechanism [16], namely it is not possible to produce a gap with one-gluon exchange if the resolution of our apparatus is infinitely good. In real life, though, there is always a finite contribution to the gap from one-gluon exchange because we cannot detect soft gluons with $p_{\perp} \lesssim \mu$. This constitutes a background to the signal we are interested in and at the hadron level may be removed by using a double NBD that fits well the high-multiplicity data [3]. In addition, the exponential in eq. (2) becomes smaller as

the rapidity interval between the partons $\Delta \eta \simeq \ln(\hat{s}/|\hat{t}|)$ grows, in qualitative agreement with the data [1, 3, 4].

The resummation of the leading virtual radiative corrections to two-gluon exchange is more problematic, because in this case the solution of the BFKL equation is well behaved only for the scattering between colorless objects [17]. For the scattering between partons, e.g. gluons, the solution [14, 15]

$$\frac{d\hat{\sigma}_{sing}}{d\hat{t}} \simeq \frac{81\pi^3 \alpha_s^4}{4\hat{t}^2} \frac{\exp\left[24\ln 2\alpha_s \ln(\hat{s}/|\hat{t}|)/\pi\right]}{\left[21\zeta(3)\alpha_s \ln(\hat{s}/|\hat{t}|)/2\right]^3},\tag{3}$$

with $\zeta(3) = 1.20206...$, is valid only in an asymptotic sense, i.e. as $\Delta \eta \to \infty$, since at large but finite $\Delta \eta$ it would exhibit infrared divergences at each order in the expansion in α_s . Leaving the theoretical details apart, eq. (3) states that even though of higher order the signal (3) quickly becomes more important than the background (2) as the gap width grows.

2.2 Gaps at the hadron level

In order to make a prediction at the hadron level, eq. (2) and (3) must be convoluted with parton densities, $f(x_{a,b}; \mu_s)$, with $x_{a,b}$ the momentum fractions of the incoming partons and $\mu_s \gg \lambda_{QCD}$ a factorization scale. Thus we may compute the dijet production rate as a function of the rapidity difference, $\Delta \eta = \eta_{j_1} - \eta_{j_2}$. In doing that, it is convenient to fix the x's, in order to minimize the variations induced by the parton densities, which have nothing to do with the parton dynamics we want to examine [14, 15]. Since $\hat{s} = x_a x_b s$ and $\Delta \eta \simeq \ln(\hat{s}/p_\perp^2)$, this implies to run $\Delta \eta$ up with the collider c.m. energy \sqrt{s} , e.g. to compare dijet production data with a rapidity gap, at fixed values of the x's and the jet transverse momenta p_\perp , in the Tevatron runs at $\sqrt{s} = 1.8$ TeV and $\sqrt{s} = 630$ GeV. Within the values of $\Delta \eta$ kinematically accessible at the Tevatron, the signal (3) is not very sensitive to variations of $\Delta \eta$ [15].

Otherwise in a data sample at fixed s one must run $\Delta \eta$ up with the x's [18]. In this case the prediction for the gap fraction as a function of the gap width shows an abrupt rise at the largest gap widths kinematically allowed. However much of it is not due to the growth of the singlet contribution in the parton dynamics, eq.(3), but merely to the

parton luminosity which as $x \to 1$ falls off faster for the inclusive dijet production than for the one with the gap ¹.

In the parton densities we choose the factorization scale $\mu_s \gg \lambda_{QCD}$ because we must allow for the emission of soft hadrons in the rescattering between spectator partons in the underlying event in accordance with the factorization theorems [20]. In the double diffraction events, it is natural to identify the scale μ_s with the threshold μ above which we see no hadrons in the rapidity gap. However in the experiments [1, 2, 3, 4] $\mu \simeq \lambda_{QCD}$, thus the factorization picture [20] does not apply, and we need a non-perturbative model that lets the gap formed at the parton level survive the rescattering between the spectator partons, which would fill the gap with soft hadrons. Using an eikonal model, Bjorken [6] estimated the rapidity-gap survival probability, $\langle |S^2| \rangle$, to be about 5-10%. Combined with the estimate $\hat{\sigma}_{sing}/\hat{\sigma}_{oct} \sim 0.1$ for the gap production rate at the parton level [6], this yields a fraction of gaps between jets at the level of 0.5-1%, which is in qualitative agreement with eq. (1).

The gap survival probability, $<|S^2|>$, deals with the low- p_{\perp} physics of the scattering between the two hadrons. It is expected then to be fairly insensitive to the gap width, since the rapidity interval between the jets $\Delta\eta$ is a kinematic parameter of the hard-interaction process. $<|S^2|>$ is expected to decrease as the s increases [6, 21], because the total cross section, σ_{tot} , is related to the area of the soft interactions, πR^2 , and to the unitarity bound by the relation, $\sigma_{tot}\simeq\pi R^2\propto\ln s^2$. Thus as s increases it is less and less likely for the two hadrons not to interact. In addition, $<|S^2|>$ is expected to grow as the momentum fraction x of the incoming partons goes to 1, because there is less and less energy available for the underlying event, i.e. for the spectator partons, in analogy with the suppression of the underlying event observed in photoproduction as $x\to 1$. Thus if data for dijet production with a gap, at fixed values of the x's and the jet transverse momenta p_{\perp} , are compared for Tevatron runs at $\sqrt{s}=1.8$ TeV and $\sqrt{s}=630$ GeV, the main change should come from $<|S^2|>$ and should show an increase in the gap fraction going from $\sqrt{s}=1.8$ TeV to $\sqrt{s}=630$ GeV.

Finally, we recall that the gap fraction, $f_g \sim 7\%$, in excess of the background at HERA [4] is much higher than the one measured at the Tevatron (1). As in $p\bar{p}$ collisions, in photoproduction in $e\,p$ collisions, a rapidity gap in dijet production may be formed via electroweak as well as two-gluon exchange. Having ruled out electroweak exchange

¹This kinematic phenomenon is exactly the reverse of the one noted for the K-factor in inclusive dijet production in ref. [19].

because it cannot account for the size of the gap fraction, two-gluon exchange occurs at $\mathcal{O}(\alpha_s^4)$ and receives contributions from resolved and direct photons. At the parton level singlet exchange in resolved-photon production is the same as in sect. 2.1, and so is the probability of producing a gap, for equal values of the gap width $\Delta \eta_c^2$; at the hadron level the gap survival probability, $\langle |S^2| \rangle$, is expected to be larger because the γp c.m. energy is only a fraction of the ep c.m. energy, which is $\sqrt{s} \simeq 300$ GeV, and because the parton densities in the photon are stiffer than the ones in the proton as $x \to 1$, and yield a larger contribution for processes where the underlying event is suppressed. The larger stiffness of the parton densities in the photon is related to the contribution from direct-photon production that beyond the lowest order in dijet production, $\mathcal{O}(\alpha_s)$, cannot be unambiguously disentangled from the resolved-photon component. The direct-photon component, having no underlying event, has $\langle |S^2| \rangle = 1$.

The discussion of the topics in this section, which reflects though the state of the art, is very hand-waving and no detailed phenomenological predictions have been attempted. The situation may be considerably improved: at the parton level by performing next-to-leading order (NLO) calculations, as we now briefly discuss; at the hadron level by dispensing with the gap survival probability as we will show in the next section.

2.3 Improving the model

The BFKL theory, through which we have derived eq. (2) and (3), is not suitable for a detailed study of jet production because within the BFKL theory the jets have no structure, i.e. are point-like. This drawback is even more acute for dijet production with a rapidity gap because as we said in the Introduction the experiments measure the gap width, $\Delta \eta_c$, between the edges of the jet cones, which differs from the rapidity difference, $\Delta \eta$, between the jet centers by the cone sizes R, $\Delta \eta_c = \Delta \eta - 2R$. The BFKL approximation is not able to distinguish between $\Delta \eta$ and $\Delta \eta_c$. In addition, as we pointed out the calculation for eq. (3) is not infrared stable.

In order to examine the gap fraction as a function of the gap width between the jetcone edges, while accounting properly for the cone structures, we need an exact higherorder calculation which includes, though, the basic features of color-singlet exchange.

²Notice, however, that the Tevatron experiments [1, 2, 3] use a cone size R = 0.7, while the ZEUS Collaboration [4] uses R = 1.

The simplest calculation of this kind for the dijet production rate with a rapidity gap is $\mathcal{O}(\alpha_s^4)$. At the moment this is unfeasible because one needs to know two-loop matrix elements, which have not been computed yet. However the gap fraction, i.e. the ratio of the rapidity-gap to the inclusive dijet production, may be computed subtracting out from the unity the ratio of the three-jet to the inclusive dijet production, for which at $\mathcal{O}(\alpha_s^4)$ we need only known one-loop and tree-level matrix elements. The preparation of an NLO three-jet Monte Carlo, which could also serve to this scope, is in progress [22]. The jet production with a gap in rapidity would then be computed by requiring that any extra partons besides the ones we tag on be emitted within the jet cones. Therefore a distinction between octet and singlet contributions would not be done. In addition, the calculation would be infrared stable, and the dependence on the factorization scale strongly reduced.

3 Rapidity-gap physics at the LHC

3.1 Higgs-boson production via weak-boson fusion

As we anticipated in the Introduction, double hard diffraction offers a test ground for the production of a Higgs boson via weak-boson fusion, WW, $ZZ \to H$, at hadron supercolliders [5, 6]. Differently from the $gg \to H$ production mechanism, Fig. 1(a), WW, $ZZ \to H$ production, Fig. 1(b), has a distinct radiation pattern with a gap in parton production in the central-rapidity region, because no color is exchanged between the quarks that emit the weak bosons. This may allow us to distinguish the WW, $ZZ \to H$ signal from the overwhelming WW QCD and $t\bar{t}$ backgrounds, Fig. 2, which would have no gaps in parton production.

As we have seen in sect. 2.2 it is not sufficient to produce a gap at the parton level, since the soft hadrons produced by the underlying event of the pp scattering usually fill the gap. This scenario applied as such at the SSC collider, and led Bjorken [6] to introduce the gap survival probability, $\langle |S^2| \rangle$. However, at the LHC collider the requirement of running at very high luminosity yields overlapping events in the same bunch crossing which are an additional source of soft hadrons and would further, and hopelessly, suppress the gap signal. A way out of this deadlock is to require a gap in

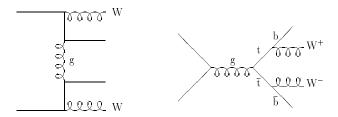


Figure 2: (a) One of the Feynman diagrams that contribute to WW QCD production, and (b) $t\bar{t}$ production.

minijet production [23, 24] rather than in soft-hadron production ³. This has the additional advantage of dispensing with the gap survival probability, because the production of soft hadrons in the rescattering of the spectator partons is clearly unrestricted. Then $\langle |S^2| \rangle = 1$, and since $\mu_s \gg \lambda_{QCD}$ there is no factorization breaking [15].

3.2 Forward-jet tagging and the minijet veto

A heavy Higgs boson decays predominantly into a pair of W or Z bosons, Fig. 1. In what follows we assume the subsequent leptonic decay of the weak bosons ⁴, and follow the outline of ref. [24]. The incoming W's or Z's tend to have a small momentum fraction, $x_{W,Z} \ll 1$, of the parent quarks, however they must be energetic enough to produce the Higgs boson, $E_{W,Z} \gtrsim m_H/2$. This implies that the outgoing quarks, that carry the momentum fraction (1-x) and usually hadronize into jets, are very energetic. In addition, the incoming-W, Z propagators yield the largest contribution when the transverse momentum, $p_{\perp j}$, of the incoming W's or Z's and of the outgoing jets is $p_{\perp j} \lesssim m_{W,Z}$. Therefore the outgoing jets tend to be energetic and to come out at a small angle, i.e. in the forward-rapidity region. Requiring then a forward-jet tagging greatly reduces the W W QCD and $t\bar{t}$ backgrounds with little loss for the signal.

Since the Higgs boson is heavy, the transverse momenta of the outgoing W's or Z's,

³A good introduction to the topic is in ref. [25].

⁴In the case of hadronic decay, the rapidity-gap window is partially colored by the decay products of the weak bosons, however if the Higgs boson is very heavy the outgoing weak bosons are far off shell and the hadronic decay products are boosted into cones of small opening angle, which should not spoil the rapidity-gap signature [6].

and hence of the leptons the W's or Z's decay into, are large. Thus the leptons usually come out in the central-rapidity region. An additional background reduction is achieved by requiring that the leptons are central and that they are separated by a large rapidity gap from the forward jets.

However, a further large background suppression, and the one which makes eventually the signal to stand out, is the very essence of the rapidity-gap physics: the signal and the background have a very different radiation pattern; namely for the background color is exchanged in the crossed channel, and so the transverse scale of the color exchange is is of the order of the hardness of the process, tipically $Q = \mathcal{O}(1 \text{ TeV})$. If we assume that the probability of parton bremsstrahlung is given by $f_s = \alpha_s \ln(Q^2/p_{\perp min}^2)$ then multiple bremsstrahlung occurs when $f_s \simeq 1$, i.e. when $p_{\perp min} \sim 50 \text{ GeV}$. Thus we expect the background to have multiple minijet emission in the 50 GeV range ⁵. On the other hand in the signal there is no color emission but in the formation of the outgoing quarks. As we have seen in the paragraph above the transverse scale associated with that is $p_{\perp j} = Q \sim 50\text{-80 GeV}$, thus for the signal multiple minijet emission is expected to occur when $p_{\perp min} \sim 5 \text{ GeV}$. Vetoing the production of minijets with, say, $p_{\perp min} \gtrsim 20 \text{ GeV}$ drastically reduces the background while affecting the signal very little.

At the LHC a gauge of the efficiency of the background-reduction techniques described above is Z+2-jet production. At the parton level the signal is given by quark scattering, $q q \to q q Z$, via γ , W, Z-boson exchange, and resembles the Higgs-boson production via weak-boson fusion described above. The background is given by Z+2-jet events in $\mathcal{O}(\alpha_s^2)$ Drell-Yan production. Considering the leptonic decay of the Z boson and implementing the forward-jet and central-lepton tagging and the minijet veto cuts as above reduces the huge background to a level below the signal [27].

4 Conclusions

We have reviewed the state of the art for the production of rapidity gaps between jets at the Tevatron and HERA colliders, which may be accounted for by assuming the exchange of a color singlet in the crossed channel, with suppressed radiation in the central-rapidity region. We have described how at LHC energies this translates into a

⁵The minijet activity may be preliminarly tested at the Tevatron, by considering the minijet emission at $\mathcal{O}(\alpha_s^3)$ in high- p_{\perp} dijet events [26].

suppression of minijet emission which may be used to enhance Higgs-boson production via electroweak-boson exchange over the dominant QCD backgrounds.

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